294. Proposed by O. L. CALLECOT, Gettysburg, S. Dak.

Find the limit of $\sum_{n=1}^{n=\infty}\frac{2(n^2+3n+3)}{n(n+1)(n+2)(n+3)}$.

Solution by H. V. SPUNAR, C. E., East Pittsburg, Pa., and J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

$$\sum_{n=1}^{n=n}\frac{2(n^2+3n+3)}{n(n+1)(n+2)(n+3)}=\sum_{n=1}^{n=n}\left[\frac{1}{n}-\frac{1}{n+1}+\frac{1}{n+2}-\frac{1}{n+3}\right]$$

$$=\Sigma\frac{1}{n}-\Sigma\frac{1}{n+1}+\Sigma\frac{1}{n+2}-\Sigma\frac{1}{n+3}$$

$$=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots.+\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\ldots.$$

$$-\frac{1}{2}-\frac{1}{3}-\frac{1}{4}-\ldots.-\frac{1}{n}-\frac{1}{n+1}-\frac{1}{n+2}-\frac{1}{n+3}-\ldots.$$

$$+\frac{1}{3}+\frac{1}{4}+\ldots.+\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\ldots.$$

$$-\frac{1}{4}-\ldots.-\frac{1}{n}-\frac{1}{n+1}-\frac{1}{n+2}-\frac{1}{n+3}-\ldots.$$

$$=1+\tfrac{1}{3}=1\tfrac{1}{3}.$$

Also solved by C. E. White and G. B. M. Zerr.

In order for the solution of this problem to be rigorous, the matter of convergency must be investigated. The equality

$$\sum_{n=1}^{n=n}\left[\frac{1}{n}-\frac{1}{n+1}+\frac{1}{n+2}-\frac{1}{n+3}\right]=\sum_{n=1}^{n=\infty}\frac{1}{n}-\sum_{n=1}^{n=\infty}\frac{1}{n-1}+\sum_{n=1}^{n=\infty}\frac{1}{n+2}-\sum_{n=1}^{n=\infty}\frac{1}{n+3}$$

assumed to hold in the above solution is not always true.

GEOMETRY.

328. Proposed by CHARLES GILPIN, JR., Philadelphia, Pa.

A sphere with the radius R is divided into two segments by a plane passed through it half way between the center and circumference. The smaller segment is divided into two parts by a plane passed through it at right angles to the base and cutting it half way between its center and circumference. Required the contents of the two parts of the segment.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa., and BENJAMIN F. FINKEL, Ph. D., Drury College, Springfield, Mo.

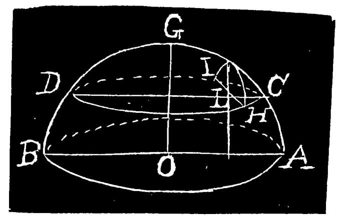
The equation of the sphere is $x^2+y^2+z^2=R^2$. The volume of $C-HIK$ is

$$V_1=\int dx \int dy \int dz =\int_{\frac14\sqrt{3}R}^{\frac12\sqrt{3}R} dx\, 2\int_0^{\sqrt{(\frac34 R^2-x^2)}} dy \int_{\frac12 R}^{\sqrt{(R^2-x^2-y^2)}} dz$$

$$=\int_{\frac14\sqrt{3}R}^{\frac12\sqrt{3}R} dx\, 2\int_0^{\sqrt{(\frac34 R^2-x^2)}} [\sqrt{(R^2-x^2-y^2)}-\tfrac12 R]\,dy$$

$$=\int_{\frac14\sqrt{3}R}^{\frac12\sqrt{3}R} 2\left[\tfrac12 y\sqrt{(R^2-x^2-y^2)}+\tfrac12(R^2-x^2)\sin^{-1}\frac{y}{\sqrt{(r^2-x^2)}}-\tfrac12 Ry\right]_0^{\sqrt{(\frac34 R^2-x^2)}} dx$$

$$=\int_{\frac14\sqrt{3}R}^{\frac12\sqrt{3}R}\left[(R^2-x^2)\cos^{-1}\left(\frac{\frac12 R}{\sqrt{(R^2-x^2)}}\right)-\tfrac12 R\sqrt{(\tfrac34 R^2-x^2)}\right]dx$$



$$=\left[(R^2x-\tfrac13 x^3)\cos^{-1}\left(\frac{\frac12 R}{\sqrt{(R^2-x^2)}}\right)\right.$$

$$\left.-\tfrac{1}{24}R\int\left(\frac{9R^4-33R^2x^2+16x^4}{(R^2-x^2)\sqrt{(\frac34 R^2-x^2)}}\,dx\right)\right]_{\frac14\sqrt{3}R}^{\frac12\sqrt{3}R}$$

$$=\left[(R^2x-\tfrac13 x^3)\cos^{-1}\frac{\frac12 R}{\sqrt{(R^2-x^2)}}\right.$$

$$\left.-\tfrac{1}{24}R\int\left(\frac{17R^2}{\sqrt{(\frac34 R^2-x^2)}}-\frac{16x^2}{\sqrt{(\frac34 R^2-x^2)}}-\frac{8R^4}{(R^2-x^2)\sqrt{(\frac34 R^2-x^2)}}\right)dx\right]_{\frac14\sqrt{3}R}^{\frac12\sqrt{3}R}$$

$$=\left[(R^2x-\tfrac13 x^3)\cos^{-1}\frac{\frac12 R}{\sqrt{(R^2-x^2)}}-\tfrac{1}{24}R\left(11R^2\sin^{-1}\left(\frac{x}{\frac12\sqrt{3R}}\right)\right.\right.$$

$$\left.\left.-16R^2\sin^{-1}\frac{x}{\sqrt{3(R^2-x^2)}}+8x\sqrt{(\tfrac34 R^2-x^2)}\right]_{\frac14\sqrt{3}R}^{\frac12\sqrt{3}R}$$

$$=R^3\left[\tfrac{13}{72}\pi-\tfrac{15}{64}\sqrt{3}\cos^{-1}\frac{2}{\sqrt{13}}-\tfrac23\sin^{-1}\frac{1}{\sqrt{13}}+\tfrac{1}{16}\sqrt{3}\right].$$

The volume of $D-HIK$ is

$$V_2=4\int_{\frac12 R}^{R} dx \int_0^{\sqrt{(R^2-x^2)}} dy \int_0^{\sqrt{(R^2-x^2-y^2)}} dx - V_1=\tfrac{5}{24}\pi R^3-V_1$$

$$=R^3\left[\frac{\pi}{36}+\tfrac{15}{64}\sqrt{3}\cos^{-1}\left(\frac{2}{\sqrt{13}}\right)+\tfrac23\sin^{-1}\left(\frac{1}{\sqrt{13}}\right)-\tfrac{1}{16}\sqrt{3}\right].$$

Also solved with different results by H. V. Spunar and J. Scheffer.